

Math 8 Honours Assignment 1.4 Prime Factorization and Number of Factors (Part 1)

1. Multiply the following without using a calculator:

a) $12 \times 16 =$

①
$$\begin{array}{|c|c|} \hline 10 & 2 \\ \hline 10 & 20 \\ \hline 6 & 12 \\ \hline \end{array} = 192 //$$

or

$$4 \times 3 \times 4 \times 4 = 64 \times 3 = 120 + 12 = 192 //$$

b) $15 \times 20 =$

$$15 \times 2 \times 10 = 30 \times 10 = 300 //$$

(Easy)

c) $9 \times 16 \times 2 =$

$$3 \times (3 \times 4 \times 4) \times 2 = 12 \times 12 \times 2 = 144 \times 2 = 288 //$$

d) $27 \times 4 =$

$$9 \times 3 \times 4 = 9 \times 12 = 108 //$$

or

$$20 + 20 = 108 //$$

e) $14 \times 21 =$

$$7 \times 2 \times 3 \times 7 = 49 \times 6 = 300 - 6 = 294 //$$

or

$$49 \times 6 = 240 + 54 = 294 //$$

f) $8 \times 12 \times 6 =$

$$= 96 \times 6 = 540 + 36 = 576 //$$

or

$$8 \times 72 = 560 + 16 = 576 //$$

g) $15 \times 12 \times 35 =$

$$3 \times 5 \times 2 \times 6 \times 35 = 10 \times 3 \times 3 \times 2 \times 35 = 10 \times 9 \times 70 = 630 //$$

h) $18 \times 14 \times 5 =$

$$9 \times 2 \times 7 \times 2 \times 5 = 63 \times 2 \times 10 = 1260 //$$

i) $15 \times 24 \times 3 =$

$$3 \times 5 \times 2 \times 12 \times 3 = 10 \times 9 \times 12 = 1080 //$$

2. Find the Prime Factorization for each of the following numbers without using a calculator:

a) 24

$$= 2^3 \times 3^1 //$$

d) 845

$$= 5 \times 169 = 5 \times 13^2 //$$

$$\begin{array}{r} 169 \\ 5 \overline{) 845} \\ \underline{34} \\ 4 \end{array} \quad \begin{array}{r} 13 \\ \times 13 \\ \hline 39 \\ 13 \\ \hline 169 \end{array}$$

g) 864

$$= 8 \times 108 = 8 \times 4 \times 27 = 2^3 \times 2^2 \times 3^3 = 2^5 \times 3^3 //$$

b) 1844

$$\begin{aligned}
 &= 922 \times 2 \\
 &= 461 \times 2 \times 2 \\
 &= 2^2 \times 461 \\
 &\quad \underline{\underline{\text{PRIME}}}
 \end{aligned}$$

e) 3844

$$\begin{aligned}
 &1922 \times 2 \\
 &= 961 \times 2 \times 2 \\
 &= 31^2 \times 2^2 //
 \end{aligned}$$

h) 5040

$$\begin{aligned}
 &= 2^4 \times 3^2 \times 5^1 \times 7^1 \\
 &\quad \begin{array}{l} 5040 \\ \swarrow \searrow \\ 504 \times 10 \\ \swarrow \searrow \quad \swarrow \searrow \\ 252 \times 2 \quad 5 \\ \swarrow \searrow \quad \swarrow \searrow \\ 126 \times 2 \quad 9 \times 7 \\ \swarrow \searrow \quad \swarrow \searrow \\ 63 \times 2 \quad 3 \times 7 \\ \swarrow \searrow \quad \swarrow \searrow \\ 9 \times 7 \end{array}
 \end{aligned}$$

c) 20124

$$\begin{aligned}
 &5031 \times 4 \\
 &\quad \begin{array}{l} 5031 \\ \swarrow \searrow \\ 503 \times 9 \\ \swarrow \searrow \\ 13 \times 43 \end{array} \\
 &= 2^2 \times 3^2 \times 13^1 \times 43^1
 \end{aligned}$$

f) 12056

$$\begin{aligned}
 &1507 \times 8 \\
 &\quad \begin{array}{l} 1507 \\ \swarrow \searrow \\ 11 \times 137 \end{array} \\
 &= 2^3 \times 11^1 \times 137 //
 \end{aligned}$$

i) 454,597

$$\begin{aligned}
 &11 \times 41327 \\
 &\quad \begin{array}{l} 41327 \\ \swarrow \searrow \\ 11 \times 3757 \\ \quad \swarrow \searrow \\ \quad 13 \times 289 \\ \quad \quad \swarrow \searrow \\ \quad \quad 17 \times 17 \end{array} \\
 &= 11^2 \times 13 \times 17^2
 \end{aligned}$$

3. Indicate which of the following are perfect squares:

a) ~~24~~, 36, 225, 169, ~~189~~, ~~224~~ _____

$$6^2 = 36$$

$$15^2 = 225$$

$$13^2 = 169$$

b) 16, ~~27~~, ~~72~~, ~~125~~, 289, 324 _____

$$4^2 = 16$$

$$17^2 = 289$$

$$18^2 = 324$$

c) ~~25~~, 900, 1024, 144, ~~54~~, ~~-64~~ _____

$$30^2 = 900$$

$$32^2 = 1024$$

$$12^2 = 144$$

4. Indicate which of the following products will be perfect squares:

a) $2^4 \times 3^2 \times 5^2 =$

b) $2^2 \times 3^4 \times 7^2 =$

c) $3^4 \times 3^3 \times 7^5 =$

d) $27 \times 24 \times 8 =$

e) $16 \times 36 \times 2 =$

f) $3 \times 12 \times 8 =$

g) $2 \times 2^2 \times 2^3 =$

h) $5 \times 5^4 \times 5^6 =$

i) $2^3 \times 3^4 \times 12^3 =$

5. Given that N is a natural number find the lowest value of N such that the square root will become a positive integer:

a) $2^3 5^1 7^2 N$

b) $4^2 7^2 5^2 N$

c) $3^4 5^3 12 N$

$$\text{d) } 38412 \times N$$

$$\text{e) } 13992 \times N$$

$$\text{f) } 664 \times (N - 1)$$

6. Given that N is an integer greater than one and $N \neq 0$, what is the lowest value of N so that " K " is a perfect square (Given $K \neq 0$)

$$\text{a) } K = N \times 3^3 \times 21$$

$$\text{d) } K = (N - 1) \times 7^7 \times 121$$

$$\text{b) } K = N \times 3^3 \times 5^5$$

$$\text{e) } K = N^2 + N$$

$$\text{c) } K = N \times 75 \times 169$$

$$\text{f) Challenge: } K = (3N - 24)(N - 28)$$

7. By looking at the prime factorization of a number, how can you determine whether if it is a perfect square or not? Explain:

8. Explain how you would multiply the following using the prime factorization. Do Not use a calculator:
 $15 \times 25 \times 35 \times 45 \times 16$

9. A building has 18 storeys above ground and 5 storeys below ground. If each storey is 5 meters high, then how tall is the building?

10. In a magic square, the numbers in each row, column, and diagonal have the same sum. This is called the magic sum. What is the magic sum for this magic square?

2	3	-2
-3	1	5
4	-1	0

b) Multiply each integer in the square by -2. Is the result another magic square? If so, what is the new magic square?

8	1	6
3	5	7
4	9	2

17	21	11	18	15
23	5	7	14	16
4	6	13	20	22
10	12	19	4	3
11	18	25	2	9

$$1 \rightarrow 25 = \frac{25(26)}{2}$$

$$\text{value of each column} = \frac{5 \times 13}{25(26)} = 65 //$$

c) Create a magic square with a magic sum of -12.

11. Find the value of "N" in each equation:

a) $18,000 = N \times 2^3 \times 5^3 \times 6$

b) $80,640 = N \times 2^5 \times 12 \times 14$

12. Let a, b, c, d, and e be distinct integers such that $(6-a)(6-b)(6-c)(6-d)(6-e) = 45$. What is the value of $a+b+c+d+e$?

$$(-1)(-3)(3)(5)(1)$$

$$\underline{(6-a)} + \underline{(6-b)} + \underline{(6-c)} + \underline{(6-d)} + \underline{(6-e)} = \underline{-1} + \underline{1} + \underline{3} + \underline{-3} + \underline{5}$$

$$30 - a - b - c - d - e = 5$$

$$30 - 5 = a + b + c + d + e$$

$$\boxed{25 = a + b + c + d + e}$$